Power spectra from an inflaton coupled to the Gauss-Bonnet term

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We consider power-law inflation with a Gauss-Bonnet correction inspired by string theory. We analyze the stability of cosmological perturbations and obtain the allowed parameter space. We find that for GB-dominated inflation ultra-violet instabilities of either scalar or tensor perturbations show up on small scales. The Gauss-Bonnet correction with a positive (or negative) coupling may lead to a reduction (or enhancement) of the tensor-to-scalar ratio in the potential-dominated case. We place tight constraints on the model parameters by making use of the WMAP 5-year data.

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I. INTRODUCTION

Inflation in the early universe has become the standard model for the generation of cosmological perturbations in the universe, the seeds for large-scale structure and temperature anisotropies of the cosmic microwave background (CMB). The simplest scenario of cosmological inflation is based on the presence of a single, minimally-coupled scalar field with a slowly varying potential. Quantum fluctuations of this inflaton field give rise to an almost scale-invariant and primordial power spectrum of isentropic perturbations (see Refs. [1, 2] for reviews).

String theory is often regarded as the leading candidate for unifying gravity with the other fundamental forces and for a quantum theory of gravity. It is known that there are correction terms of higher orders in the curvature to the lowest order effective supergravity action coming from superstrings, which may play a significant role in the early universe. The simplest such correction is the Gauss-Bonnet (GB) term in the low-energy effective action of the heterotic string [3]. Such a term provides the possibility of avoiding the initial singularity of the universe [4]. In the presence of an exponential potential for the modulus field, nonsingular cosmological solutions were found which begin in an asymptotically flat region, undergo super-exponential inflation and end with a graceful exit to a phase with decreasing Hubble radius [5].

There are many works discussing accelerating cosmology with the GB correction in four and higher dimensions [6–9]. Recently it has been shown that the GB term might give rise to violent instabilities of tensor perturbations [10]. A model in which inflation is driven by the GB term and a higher-order kinetic energy term was studied. When the GB term dominates the dynamics of the background, tensor perturbations exhibit violent negative instabilities around a de Sitter background on small scales, in spite of the fact that scale-invariant scalar perturbations can be achieved [10]. Besides the kinetic and GB terms, a scalar potential arises naturally from supersymmetry breaking or other non-perturbative effects. So far, the inflationary solutions and the resulting cosmological perturbations have not been studied in detail when both the GB correction and the scalar potential are present.

In this work we investigate single-field inflation with a non-minimal coupling of the inflaton to the GB term. We confront the predictions for the primordial power spectra for scalar and tensor modes with WMAP data. In doing so, we restrict our attention to power-law inflation, which is realized when both the potential and the scalar-GB coupling take an exponential form. When the potential is dominant, we find that it is possible to realize observationally supported density perturbations. When the GB term is dominant, either tensor or scalar perturbations exhibit negative instabilities on small scales, which invalidate the assumption of linear perturbations.

This paper is organised as follows. In section II we calculate the power spectra of scalar and tensor perturbations for an inflaton that is coupled to the GB term. We restrict our analysis to situations in which the mode equations are solved by Bessel functions. In section III we study the case of power-law inflation in detail. Section IV is devoted to conclusions.

II. COSMOLOGICAL PERTURBATIONS

We consider the following action

$$S=\int d^4x\sqrt{g}\left[\frac{1}{2}R-\frac{\omega}{2}(\nabla\phi)^2-V(\phi)-\frac{1}{2}\xi(\phi)R^2_{\rm GB}\right],\quad(1)$$

where ϕ is a scalar field with a potential $V(\phi)$, $\omega=\pm1$, R denotes the Ricci scalar, and $R^2_{\rm GB}=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2$ is the GB term. We work in Planckian units, $\hbar=c=8\pi G=1$. In a spatially flat Friedmann-Robertson-Walker (FRW) universe with scale factor a,

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the background equations read

$$6H^2 = \omega\dot{\phi}^2 + 2V + 24\dot{\xi}H^3, \tag{2}$$
$$2\dot{H} = -\omega\dot{\phi}^2 + 4\ddot{\xi}H^2 + 4\dot{\xi}H(2\dot{H} - H^2), \tag{3}$$

where a dot represents the time derivative and $H = \dot{a}/a$ denotes the expansion rate. To compare the contributions from the potential and the GB term, we use the ratio of the second to the third term on the right-hand side of Eq. (2), $\lambda = V/12\dot{\xi}H^3$. For a potential-dominated model $|\lambda| > 1$, for a GB-dominated model $|\lambda| < 1$.

At linear order in perturbation theory, the Fourier modes of curvature perturbations satisfy [11]

$$v'' + \left(c_{\mathcal{R}}^2 k^2 - \frac{z_{\mathcal{R}}''}{z_{\mathcal{R}}}\right) v = 0, \tag{4}$$

where a prime represents a derivative with respect to conformal time $\tau = \int a^{-1}dt$, and

$$z_{\mathcal{R}}^2 = \frac{a^2(\omega\dot{\phi}^2 - 12I\dot{\xi}H^2)}{(H+I)^2}, \tag{5}$$
$$c_{\mathcal{R}}^2 = 1 + \frac{-16I\dot{\xi}\dot{H} + 8I^2(\ddot{\xi} - \dot{\xi}H)}{\omega\dot{\phi}^2 - 12I\dot{\xi}H^2}, \tag{6}$$

with $I = -2\dot{\xi}H^2/(1 - 4\dot{\xi}H)$.

If we can write $z_{\mathcal{R}} = Q_{\mathcal{R}}|\tau|^{1/2-\nu_{\mathcal{R}}}$, with $Q_{\mathcal{R}}$ and $\nu_{\mathcal{R}}$ constant, we find $z_{\mathcal{R}}''/z_{\mathcal{R}} = (\nu_{\mathcal{R}}^2 - 1/4)/\tau^2$. Additionally, if $c_{\mathcal{R}}^2$ is a positive constant, the general solution of Eq. (4) is a linear combination of Hankel functions

$$v = \frac{\sqrt{\pi|\tau|}}{2} e^{i(1+2\nu_{\mathcal{R}})\pi/4}\left[c_1 H_{\nu_{\mathcal{R}}}^{(1)}(c_{\mathcal{R}}k|\tau|) + c_2 H_{\nu_{\mathcal{R}}}^{(2)}(c_{\mathcal{R}}k|\tau|)\right]. \tag{7}$$

We choose $c_1 = 0$ and $c_2 = 1$, so that positive frequency solutions in the Minkowski vacuum are recovered in an asymptotic past. Since $H_{\nu_{\mathcal{R}}}^{(2)}(c_{\mathcal{R}}k|\tau|) \to (i/\pi)\Gamma(\nu_{\mathcal{R}})(c_{\mathcal{R}}k|\tau|/2)^{-\nu_{\mathcal{R}}}$ for long wavelength perturbations ($c_{\mathcal{R}}k|\tau| \ll 1$), the curvature perturbation after crossing of the Hubble radius is given by

$$\mathcal{R} = \frac{v}{z_{\mathcal{R}}} = e^{i(3+2\nu_{\mathcal{R}})\pi/4}\frac{c_{\mathcal{R}}^{-\nu_{\mathcal{R}}}}{4Q_{\mathcal{R}}}\frac{\Gamma(\nu_{\mathcal{R}})}{\Gamma(3/2)}\left(\frac{k}{2}\right)^{-\nu_{\mathcal{R}}}. \tag{8}$$

The power spectrum of curvature perturbations, $\mathcal{P}_{\mathcal{R}} = k^3|\mathcal{R}|^2/2\pi^2$, becomes

$$\mathcal{P}_{\mathcal{R}} = \frac{c_{\mathcal{R}}^{-2\nu_{\mathcal{R}}}}{4\pi^2 Q_{\mathcal{R}}^2}\left(\frac{\Gamma(\nu_{\mathcal{R}})}{\Gamma(3/2)}\right)^2\left(\frac{k}{2}\right)^{3-2\nu_{\mathcal{R}}}, \tag{9}$$

with spectral index

$$n_{\mathcal{R}} - 1 = 3 - 2\nu_{\mathcal{R}}. \tag{10}$$

The Fourier modes of tensor perturbations satisfy [11]

$$u'' + \left(c_T^2 k^2 - \frac{z_T''}{z_T}\right) u = 0, \tag{11}$$

where

$$z_T^2 = a^2(1 - 4\dot{\xi}H), \tag{12}$$
$$c_T^2 = 1 - \frac{4(\ddot{\xi} - \dot{\xi}H)}{1 - 4\dot{\xi}H}. \tag{13}$$

As above, if c_T^2 is a positive constant and if z_T can be written as $z_T = Q_T|\tau|^{1/2-\nu_T}$ with constant Q_T and ν_T, the power spectrum of tensor perturbations, $\mathcal{P}_T = 2k^3|2u/z_T|^2/2\pi^2$, is given by

$$\mathcal{P}_T = \frac{8c_T^{-2\nu_T}}{4\pi^2 Q_T^2}\left(\frac{\Gamma(\nu_T)}{\Gamma(3/2)}\right)^2\left(\frac{k}{2}\right)^{3-2\nu_T}. \tag{14}$$

The spectral index of the gravitational wave power spectrum is

$$n_T = 3 - 2\nu_T. \tag{15}$$

An important observational quantity is the tensor-to-scalar ratio,

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = 8\frac{Q_{\mathcal{R}}^2}{Q_T^2}\frac{c_{\mathcal{R}}^{2\nu_{\mathcal{R}}}}{c_T^{2\nu_T}}\left(\frac{\Gamma(\nu_T)}{\Gamma(\nu_{\mathcal{R}})}\right)^2\left(\frac{k}{2}\right)^{2\nu_{\mathcal{R}}-2\nu_T}. \tag{16}$$

III. POWER-LAW INFLATION

In this section we consider power-law solutions given by

$$a \propto t^{1/\gamma}, \quad H = \frac{1}{\gamma t}. \tag{17}$$

The power-law solution that involves a single power of cosmic time given by (17) is

$$\dot{\xi} = \alpha t, \quad V = \frac{\beta}{\gamma t^2}, \tag{18}$$

where α and β are constants, which characterize the contributions from the GB term and the potential, respectively. In what follows we restrict our attention to a positive scalar potential, i.e., $\beta > 0$. The case of $\alpha < 0$ is permitted as long as the total energy density is positive. In this case the potential-to-GB ratio, $\lambda = \gamma^2\beta/12\alpha$, becomes a constant. Eliminating the $\omega\dot{\phi}^2$ term from Eqs. (2) and (3), we have

$$(\beta+1)\gamma^2 - (2\alpha+3)\gamma + 10\alpha = 0, \tag{19}$$

which has two solutions, γ_+ and γ_-,

$$\gamma_\pm = \frac{(2\alpha+3) \pm \sqrt{(2\alpha+3)^2 - 40\alpha(\beta+1)}}{2(\beta+1)}. \tag{20}$$

An inflationary solution ($\ddot{a} > 0$) is obtained for $0 < \gamma < 1$. The sign of ω depends on the sign of $(\gamma^2 - 2\alpha\gamma - 2\alpha)$. In the case $\alpha \approx 0$ (no GB contribution), we choose $\omega = 1$

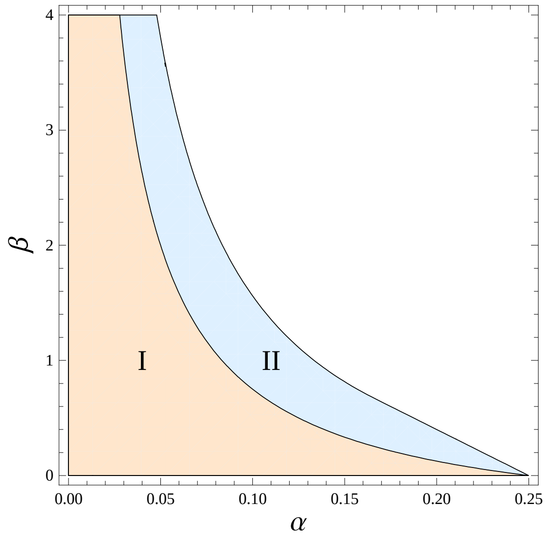

FIG. 1: Parameter space (α, β) for inflationary solutions with $0 < \gamma_- < 1$. In region I, tensor modes are unstable on small scales, while scalar modes are stable. The opposite holds in region II.

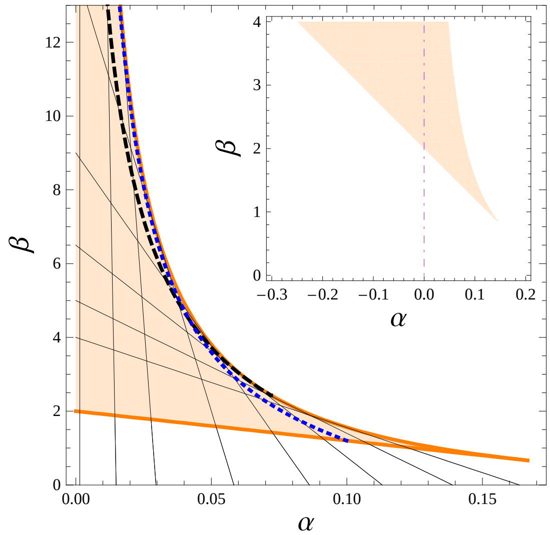

FIG. 2: Parameter space (α, β) for inflationary solutions with $0 < \gamma_+ < 1$. In the shaded region both scalar and tensor modes are stable on small scales. Solid lines correspond to $\gamma = 0.005,\ 0.05,\ 0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6$ (from bottom left to right). Models with $\omega = -1$ ($\omega = 1$) are above (below) the black dashed line. The blue dotted line represents the potential-GB equality ($\lambda = 1$). The potential-dominated (GB-dominated) models are below (above) this line.

and $\gamma = \gamma_+$, since $\gamma_- = 0$. The condition for inflation requires $\beta > 2$. In the case $\beta = 0$ (no potential), we choose $\omega = -1$ and $\gamma = \gamma_-$, since $\gamma_+ > 1$. The condition for inflation requires $\alpha < 1/4$ [10]. Taking the positive sign of ϕ, we obtain

$$\phi = \frac{1}{\gamma}\sqrt{\frac{2|\gamma^2 - 2\alpha\gamma - 2\alpha|}{\gamma}}\ln t, \tag{21}$$

$$\xi(\phi) = \frac{\alpha}{2}\exp\left(\sqrt{\frac{2\gamma}{|\gamma^2 - 2\alpha\gamma - 2\alpha|}}\,\gamma\phi\right), \tag{22}$$

$$V(\phi) = \frac{\beta}{\gamma}\exp\left(-\sqrt{\frac{2\gamma}{|\gamma^2 - 2\alpha\gamma - 2\alpha|}}\,\gamma\phi\right), \tag{23}$$

which indicate that an exponential potential and an exponential coupling give rise to exact power-law solutions.

Let us derive the spectral indices of scalar and tensor perturbations. From Eqs. (6) and (10) for the scalar perturbations we find

$$c_{\mathcal{R}}^2 = 1 + \frac{16\alpha^2(-\gamma^2 + 5\alpha\gamma - \alpha)}{[(\gamma^2 - 2\alpha\gamma - 2\alpha)(\gamma - 4\alpha) + 12\alpha^2](\gamma - 4\alpha)}, \tag{24}$$

$$n_{\mathcal{R}} - 1 = 3 - \left|\frac{3-\gamma}{1-\gamma}\right|, \tag{25}$$

which implies a red spectrum for $0 < \gamma < 1$. As the phase speed (speed of sound) is constant for power-law inflation, the result for the power spectrum obtained above applies.

From Eqs. (13) and (15) we find for the tensor perturbations

$$c_T^2 = 1 - \frac{4\alpha(\gamma - 1)}{\gamma - 4\alpha}, \tag{26}$$

$$n_T = 3 - \left|\frac{3-\gamma}{1-\gamma}\right|, \tag{27}$$

the same spectral index as for the scalar perturbations. The tensor-to-scalar ratio (16) reads

$$r = 16\frac{(\gamma^2 - 2\alpha\gamma - 2\alpha)(\gamma - 4\alpha) + 12\alpha^2}{(\gamma - 6\alpha)^2}\left(\frac{c_{\mathcal{R}}^2}{c_T^2}\right)^{\nu_{\mathcal{R}}}. \tag{28}$$

Note that the tensor-to-scalar ratio is independent of k, as in minimally coupled power-law models. However, the phase speed of scalar and tensor perturbations is different from the speed of light in general, e.g. for the case $1/4 > \alpha \gg \gamma \approx 0$ we find $c_R \approx \sqrt{6/5}$ and $c_T \approx 0$.

If $\alpha = 0$, we have $c_{\mathcal{R}}^2 = c_T^2 = 1$. If on top $0 < \gamma \ll 1$, $n_{\mathcal{R}} - 1 = n_T \approx -2\gamma$, which yield nearly scale-invariant scalar and tensor perturbations, and $r = -8n_T$ — the standard consistency relation [2] for single-field slow-roll inflation. If $\beta = 0$ and $0 < \gamma < 1$, it was shown that the tensor perturbations suffer from instabilities on small scales, because $c_T^2 = 2\gamma - 5$ is negative [10]. This type of instabilities has been also found in Refs. [12].

In Figure 1 we show the parameter space (α, β) for inflationary solutions with $0 < \gamma_- < 1$. In region I, scalar modes are stable while tensor modes are unstable on small scales ($c_{\mathcal{R}}^2 > 0$, but $c_T^2 < 0$). In region II, tensor modes are stable, while scalar modes are unstable

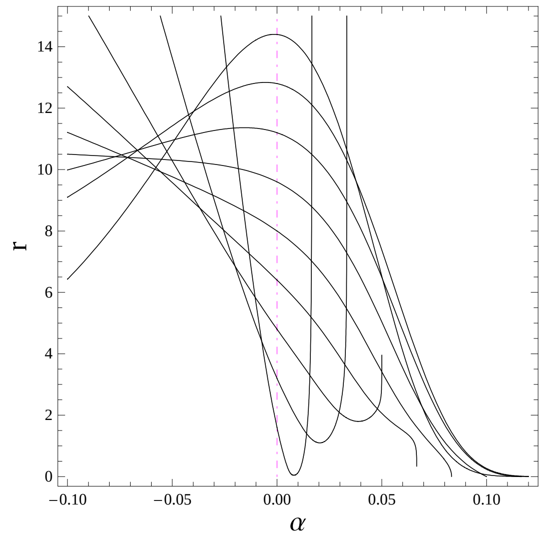

FIG. 3: Tensor-to-scalar ratio r versus GB-coupling parameter α for inflationary solutions with $\gamma = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 (from bottom to top along the dash-dotted line).

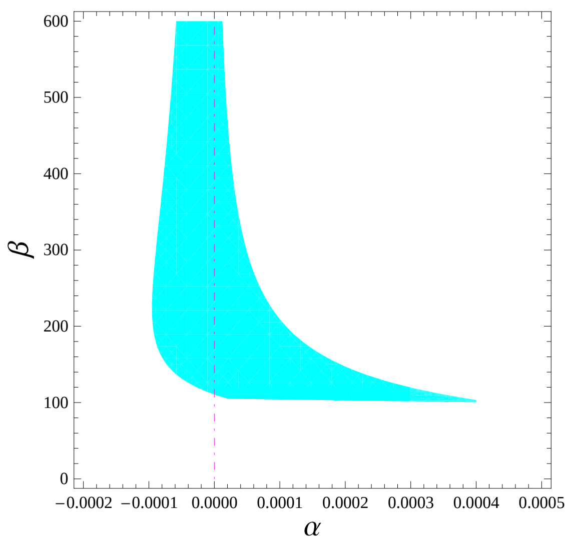

FIG. 4: Observational constraints on the inflation-model parameters from the WMAP 5-year analysis. In the shaded region, $0.942 < n_{\mathcal{R}} < 1$ and $0 < r < 0.43$.

on small scales, since now $c_T^2 > 0$ and $c_{\mathcal{R}}^2 < 0$. Unless the system is initially in a Minkowski vacuum state (in which both $c_{\mathcal{R}}^2$ and c_T^2 are positive) before the GB correction becomes important, a problem arises when we quantize scalar and tensor modes. We conclude that all solutions involving γ_- are excluded by stability arguments.

In Figure 2 we show the parameter space (α, β) for inflationary solutions with $0 < \gamma_+ < 1$. Now, both $c_{\mathcal{R}}^2$ and c_T^2 are positive. We show the potential-dominated (GB-dominated) region with $\omega = 1$ ($\omega = -1$). Most of the shaded region corresponds to the potential-dominated inflationary solution with $\omega = 1$.

As shown in Figure 3, for inflationary solutions with $\gamma > 0.5$, there is a small range with $\alpha > 0.08$ in which the tensor-to-scalar ratio is compatible with observations. However, in that case the power spectra of curvature and tensor perturbations are far from scale-invariant. In order to obtain nearly scale-invariant spectra of density perturbations, $\gamma \ll 1$ is required from (25) and (27). This can be realized when the potential term is dominant (see also Figure 2). As we see from Fig. 3, at small values of $|\alpha|$, the GB correction leads to a reduction of the tensor-to-scalar ratio if $\alpha > 0$, while it is enhanced for $\alpha < 0$.

The recent publication of data from the Wilkinson Microwave Anisotropy Probe (WMAP) [13] has brought the global cosmological dataset to a precision where it seriously constrains inflationary models. We place constraints on the model parameters by using the results of the WMAP team. Fitting the concordance model with power-law spectra for scalar and tensor perturbations to WMAP 5-year data alone, yields $n_{\mathcal{R}} = 0.986 \pm 0.022$ (68% CL) and $r < 0.43$ (95% CL). Figure 4 shows the observational constraints from WMAP 5-year data on the parameters (α and β) of power-law inflation with GB term. In the shaded region, $0.942 < n_{\mathcal{R}} < 1$ and $0 < r < 0.43$ are assumed.

We see that WMAP data allow us to obtain rather tight constraints on the magnitude of a possible non-minimal coupling of the inflaton to the GB term ($-1 \times 10^{-4} \lesssim \alpha \lesssim 4\times10^{-4}$ from WMAP, while $\alpha < 1/4$ is possible in principle) because the constraint, $0.942 < n_{\mathcal{R}} < 1$, requires that $\gamma < 0.028$ from Eq.(25). In this range the tensor-to-scalar ratio becomes sensitive to the parameter α as shown in Figure 3.

IV. CONCLUSIONS

We have studied inflationary solutions with a non-minimally coupled Gauss-Bonnet term. We find that power-law solutions, giving rise to an exponential potential and an exponential GB coupling, are in agreement with observation. The important quantities, directly linked to observations, are the spectral indices $n_{\mathcal{R}}$ and n_T, together with the tensor-to-scalar ratio r. Provided that $c_{\mathcal{R}}^2$ and c_T^2 are positive constants in the mode equations, we obtain the spectral indices of scalar and tensor perturbations given by Eqs. (25) and (27), respectively, and the tensor-to-scalar ratio (28).

Given the model parameter α and β, there are in general two solutions, γ_- and γ_+. Although scale-invariant spectra are generated for density perturbations, either scalar or tensor perturbations are faced with ultraviolet instabilities associated with negative $c_{\mathcal{R}}^2$ or c_T^2 in the shaded region of Figure 1. In the shaded region of Figure 2, this type of instabilities can be avoided. When the potential dominates, it is possible to generate nearly scale-invariant curvature perturbations with a reduced or enhanced tensor-to-scalar ratio in the case of $\alpha > 0$ or

$\alpha < 0$, respectively.

In this work we restricted our attention to power-law solutions, which is intended to be a first step towards a more complete understanding of the influence of a non-minimal coupling to the Gauss-Bonnett term. The comparison to WMAP data showed that the cosmic microwave sky provides us with a mean to strongly constrain the magnitude of that coupling. The next step will be the investigation of the slow-roll regime for an arbitrary form of the coupling and an arbitrary potential.

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[1] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rept. **215**, 203 (1992); D. H. Lyth and A. Riotto, Phys. Rept. **314**, 1 (1999) [arXiv:hep-ph/9807278]; K. A. Malik and D. Wands, arXiv:0809.4944.

[2] J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro and M. Abney, Rev. Mod. Phys. **69**, 373 (1997) [arXiv:astro-ph/9508078]; B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. **78**, 537 (2006) [arXiv:astro-ph/0507632].

[3] D. J. Grossa and J. H. Sloana, Nucl. Phys. B **291**, 41 (1987); M. Gasperini, M. Maggiore and G. Veneziano, Nucl. Phys. B **494**, 315 (1997) [arXiv:hep-th/9611039].

[4] I. Antoniadis, J. Rizos and K. Tamvakis, Nucl. Phys. B **415**, 497 (1994) [arXiv:hep-th/9305025]; S. Kawai, M. a. Sakagami and J. Soda, Phys. Lett. B **437**, 284 (1998) [arXiv:gr-qc/9802033]; S. Kawai and J. Soda, Phys. Lett. B **460**, 41 (1999) [arXiv:gr-qc/9903017]; S. Tsujikawa, Phys. Lett. B **526**, 179 (2002) [arXiv:gr-qc/0110124]; A. Toporensky and S. Tsujikawa, Phys. Rev. D **65**, 123509 (2002) [arXiv:gr-qc/0202067].

[5] S. Tsujikawa, R. Brandenberger and F. Finelli, Phys. Rev. D **66**, 083513 (2002) [arXiv:hep-th/0207228].

[6] K. Bamba, Z. K. Guo and N. Ohta, Prog. Theor. Phys. **118**, 879 (2007) [arXiv:0707.4334]; K. Andrew, B. Bolen and C. A. Middleton, Gen. Rel. Grav. **39**, 2061 (2007) [arXiv:0708.0373]; R. Chingangbam, M. Sami, P. V. Tretyakov and A. V. Toporensky, Phys. Lett. B **661**, 162 (2008) [arXiv:0711.2122]; I. V. Kirnos and A. N. Makarenko, arXiv:0903.0083.

[7] M. Satoh, S. Kanno and J. Soda, Phys. Rev. D **77**, 023526 (2008) [arXiv:0706.3585]; M. Satoh and J. Soda, JCAP **0809**, 019 (2008) [arXiv:0806.4594]; T. Matsuda, arXiv:0906.0643.

[8] S. Nojiri, S. D. Odintsov and M. Sasaki, Phys. Rev. D **71**, 123509 (2005) [arXiv:hep-th/0504052]; G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov and S. Zerbini, Phys. Rev. D **73**, 084007 (2006) [arXiv:hep-th/0601008]; T. Koivisto and D. F. Mota, Phys. Lett. B **644**, 104 (2007) [astro-ph/0606078]; S. Tsujikawa and M. Sami, JCAP **0701**, 006 (2007) [arXiv:hep-th/0608178]; T. Koivisto and D. F. Mota, Phys. Rev. D **75**, 023518 (2007) [hep-th/0609155]; B. M. Leith and I. P. Neupane, JCAP **0705**, 019 (2007) [arXiv:hep-th/0702002]; B. C. Paul and S. Ghose, arXiv:0809.4131; M. R. Setare and E. N. Saridakis, Phys. Lett. B **670**, 1 (2008) [arXiv:0810.3296]; J. Sadeghi, M. R. Setare and A. Banijamali, arXiv:0905.1468; J. Sadeghi, M. R. Setare and A. Banijamali, arXiv:0906.0713.

[9] R. A. Brown, R. Maartens, E. Papantonopoulos and V. Zamarias, JCAP **0511**, 008 (2005) [arXiv:gr-qc/0508116]; J. H. He, B. Wang and E. Papantonopoulos, Phys. Lett. B **654**, 133 (2007) [arXiv:0707.1180]; E. N. Saridakis, Phys. Lett. B **661**, 335 (2008) [arXiv:0712.3806].

[10] Z. K. Guo, N. Ohta and S. Tsujikawa, Phys. Rev. D **75**, 023520 (2007) [arXiv:hep-th/0610336].

[11] J. Hwang and H. Noh, Phys. Rev. D **61**, 043511 (2000) [arXiv:astro-ph/9909480]; C. Cartier, J. Hwang and E. J. Copeland, Phys. Rev. D **64**, 103504 (2001) [arXiv:astro-ph/0106197]; J. Hwang and H. Noh, Phys. Rev. D **71**, 063536 (2005) [arXiv:gr-qc/0412126].

[12] G. Calcagni, B. de Carlos and A. De Felice, Nucl. Phys. B **752**, 404 (2006) [arXiv:hep-th/0604201]; A. De Felice, M. Hindmarsh and M. Trodden, JCAP **0608**, 005 (2006) [arXiv:astro-ph/0604154].

[13] E. Komatsu, *et al.*, Astrophys. J. Suppl. **180**, 330 (2009) [arXiv:arXiv:0803.0547].